NUMERICAL SIMULATIONS OF NATURAL CONVECTION HEAT TRANSFER ALONG A VERTICAL CYLINDER

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ABSTRACT

Purpose- In this article, we present a numerical solution for the problem of steady laminar flow and heat transfer characteristics of viscous incompressible fluid. **Design/methodology/approach** - For this purpose a two dimensional code has been developed to simulate the natural convection heat transfer along a vertical cylinder, for four different geometries; (i) from vertical cylinder in infinite medium (ii) from a vertical flat plate in an infinite medium (iii) from an open assembly of a finite vertical cylinder and (iv) from an open rectangular pitch assembly of cylinders.
Findings - The effects of various parameters of interest have been discussed through simulations. The Nusselt numbers of constant wall temperature and constant heat flux cylinders calculated numerically and compared with Lee et al. [1] and Hackle et al. [2] respectively and are found within reasonable agreement. For large radius, a vertical cylinder has been treated as a vertical flat plate, so that the curvature effects become negligible. For the case of vertical flat plate, Nusselt number has been compared with analytical relation for the local Nusselt number given by Jaluria [3].
Research limitations/implications - The numerical results for the local Nusselt number presented here are differing from analytical results by less than 5%.
Practical implications - The natural convection has been studied for four different geometries: The flow regime in all the case studies has been assumed to be Laminar. **Originality/value** – Computer code developed for current study can be applied to much other geometries to simulate natural convection heat transfer.
Keywords: numerical simulations; *natural convection; heat transfer; vertical cylinder; viscous fluid*

1 INTRODUCTION

The equations which govern the flow of Newtonian fluid are the Navier-Stokes equations. These equations are highly non-linear partial differential equations and known exact solutions are few in number. Exact solutions are very important not only because they are solutions of some fundamental flows but also because they serve as accuracy checks for experimental, numerical and asymptotic methods. Although computer techniques make the complete numerical integration of the Navier-Stoke equations feasible, the accuracy of the results can be established by comparison with an exact solution. The description of flow and heat transfer of a viscous fluid has many important applications in manufacturing processes in industry. Natural convection heat transfers, for a variety of cases covering various geometries and thermal conditions, have been carried out in different parts of world. A classification of the literature on natural convection heat transfer is made either on the basis of problem geometry or the method of study. In the current study, we present natural convection through cylindrical objects for different geometries. Cylindrical objects are commonly encountered in industry e.g. pipes, heater rods, coils, wires etc. The cylinders can be horizontal vertical or inclined at some angle. The nature of natural convection phenomena is quite different for these orientations. For a horizontal cylinder in infinite medium the problem of natural convection has been solved analytically as well as experimentally and numerically by Eckert [4] and Ozisik [5]. A major simplification possible in the case of natural convection from horizontal cylinder is that the problem can be specified in two dimensions, which makes the solution much easier. The same advantage can also be realized in the case of natural convection from a vertical cylinder in infinite medium. Axisymmetric solutions to this problem can be found in literature for some of the thermal boundary conditions. Natural convection heat transfer coefficients for axisymmetric vertical cylindrical geometry have been determined experimentally by Nagendra et al. [6], McEligot et al. [7] Keyhani et al. [8] and computationally by Rogers et al. [9], Schneider et al. [10], Khouaja et al. [11], Valusami et al. [12], and Jaluria, [3]. For the case of a thin vertical cylinder, like the fuel pin of PARR-2, only one numerical study carried out by Lee et al. [1] has been found in the literature that requires further validation.

Badruddin et al. [13] studied the heat transfer by natural convection through a vertical annulus embedded in porous medium. Bassam et al. [14] studied the optimized use of baffles for reduced natural convection heat transfer from a horizontal cylinder. Numerical study of the laminar natural convection flow around an array of two horizontal isothermal cylinders has been carried out by Chouikh et al. [15]. Yousefi et al. [16] carried out an experimental study of natural convection heat transfer from vertical array of isothermal horizontal elliptic cylinders. Cheng [17] has studied free convection heat and mass transfer from a horizontal cylinder of elliptic cross section in micro polar fluids. Saada et al. [18] studied the natural convection around the horizontal solid cylinder wrapped with a layer of fibrous or porous material. Ishak et al. [19] have studied the effects of transpiration on the boundary layer flow and heat transfer over the vertical slender cylinder. Basit et al. [20] have studied a computer simulation of natural convection heat transfer from an assembly of vertical cylinders of PARR-2(Pakistan Research Reactor 2). In short various researchers explored the field while considering different geometries and methods.

The main aim of this paper is to carry out numerical simulations of the natural convection heat transfer along a vertical cylinder. For this purpose the physical phenomenon of natural convection heat transfer have been described mathematically. The resulting mathematical equations are then solved using a numerical technique. The fluid motion that is produced by temperature induced buoyancy forces characterizes free or natural convection flows. Since the temperature gradients are not very large, so the natural convection flow velocities are quite small (~0.01 m/s). Therefore, the flow has been considered as laminar for the current study.

2 MATHEMATICAL DESCRIPTIONS

The convective heat transfer phenomenon, in general, can be represented by the continuity, momentum and energy balance equations in their generalized form for multiphase and multi-component transport. However, for natural convection problems modeled in the current study, only a simplified form of these equations is sufficient.

The fluid velocities are very small therefore; the compressive work and viscous dissipation terms in the energy balance equation can be neglected. The three governing equations in the cylindrical co-ordinates, for natural convection, are given as follows:

$$\frac{\partial}{\partial z}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{1}$$

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial r} = \frac{\upsilon}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) + g\beta(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) \tag{3}$$

Where

r = Radius of cylinder

u = Component of velocity in z direction

v = Component of velocity in r direction

T = Temperature

ν= Kinematics viscosity of fluid

α = Thermal diffusivity of fluid

T_∞ = Bulk temperature of the fluid

g = Gravitational acceleration

β = Volumetric coefficient of thermal expansion

The above equations are for the time independent flow of incompressible Newtonian fluids. In a natural convection problem the main driving force is the density variation in the fluid. The density is usually assumed to be a function of temperature only. Hence, the knowledge of temperature field is necessary to solve velocity field. The momentum and energy equations therefore need to be solved simultaneously. Due to this complexity, the full set of Eqs. 1-3 is quite difficult to solve, even numerically. In the present study, the full set of Eqs. 1-3 have been solved by numerical techniques, employing the finite volume method. The problem of variable density has been handled by the so-called Boussinesq approximation.

2.1 Computational Technique

Due to the non-linear and implicit nature of the above mentioned conservation equations; the analytical solutions for the above equations do not exist. Many numerical techniques are being used for the computational analyses of fluid dynamics and heat transfer problems, for example, the Finite Difference Method, the Finite Element Method, the Boundary Element Method and the Finite Volume Method. In current study finite volume method has been adopted.

Following steps in sequence have been carried out,

1. Formal integration of differential equations of fluid flow over all the (finite) control volumes of the solution domain have been carried out.
2. The governing equations (energy and momentum equations) of the system have been discritized. Discretization involves the substitution of variety of finite difference type approximations for the terms in the integrated equation representing flow processes such as convection, diffusion and sources. This converts the integral equations into system of algebraic equations.

3. Solution of algebraic equations by an iterative method (SOR, Successive over Relaxation method) have been carried out.

3 NATURAL CONVECTION ALONG VERTICAL CYLINDER IN INFINITE MEDIUM (CASE 1)

In current study, emphasis has been given on thin cylinder. The length of the cylinder has been taken to be 0.4m. Inner radius and outer radius has been taken to be 0.01m and 0.045m respectively. The heat transfer medium selected in this case is air at atmospheric pressure. The problem has been solved by considering the constant temperature boundary condition at the wall of the cylinder. The cylinder has been assumed to be surrounded by an infinite medium. The schematic diagram of this cylinder is shown in Fig.1.

3.1.1 SOLUTION METHODOLOGY

For the purpose of manual calculation, at first 4x4 grid has been considered and z-momentum equation has been solved over the domain to get the u-velocity profile. Initially assuming a supposed value of temperature for whole of the domain, the momentum equation has been solved on a staggered grid. Once the u-velocity has been obtained at all the main control volume faces, then 2D continuity equation has been solved in cylindrical coordinates to obtain the v-velocity in the domain. The continuity equation has been solved considering the main control volumes, that for temperature. So the v-velocity has been obtained at the main control volume faces. When both u- and v-velocities has been obtained then the 4x4 grid was solved for temperature profile. The temperature values at various grid points have been obtained by solving the energy equation in cylindrical coordinates.

Both momentum and energy equations are solved using the "Power Law Scheme". The wall of the cylinder has been assumed to be at 313K and the temperature of the surrounding air is taken as 293K. The difference in the wall and the surrounding temperatures appeared as the momentum source. A mesh size of 20x20 was selected to solve the problem with the given data using the computer program. This mesh has been taken to be in the domain over the cylinder, from where the air is passing.

3.1.1.1 BOUNDARY CONDITIONS

In current study, a constant temperature boundary condition has been used to numerically model the natural convection heat transfer over a vertical cylinder. The temperature of the cylinder is assumed 313 K and that of the surrounding fluid to be 293 K. The bottom end of the domain is provided constant pressure boundary to allow entrainment of fluid from it. All the physical properties of the fluid are taken to be constant with temperature. No slip boundary condition has been considered at the wall of the cylinder. Therefore, at the wall both u- and v- velocities are taken as zero.

3.1.2 RESULTS AND DISCUSSION

The axial velocity has been obtained by solving the z-momentum equation in the cylindrical coordinates. In Fig.2, it can be seen that u-velocity component is zero both at the wall and the edge of the velocity boundary layer, but has a peak inside the boundary layer. Near the wall the temperature gradient is steep, resulting in the rapid change in the density of the fluid and hence the buoyancy force is maximum near the wall but here the velocity is small due to wall effects but at some suitable distance from the wall, due to the buoyancy, the velocity rises to a maximum value and afterwards due to decrease in temperature, the density variations and hence in the buoyancy force become less effective. Therefore, the velocity of the fluid decreases and gradually goes to the free stream value, which is zero for the case of natural convection. In Fig. 2, the velocity profiles are shown at different z, where z is arbitrary distance along the length of the cylinder. These velocity profiles at different z shows that as z increases in the positive z direction the velocity profile get thicker and thicker in the radial direction. The reason for this is that as the cold fluid enters in the region of cylinder wall say at z =0, the fluid takes heat from the wall. As a result the density of the fluid decreases. This decrease in density will cause the hot fluid to rise up. Therefore, in this way as the fluid moves along the wall its u velocity increases at the center of radial distance.

In Fig. 3, the temperature profiles at various z along the cylinder are shown. These profiles shows that, as the 'r' distance increases the temperature decreases. In addition, it can be observed that the value of the temperature at corresponding nodes increases along the length of the cylinder, reason being the increase in the time of contact of the fluid with the hot wall. Fig.4 shows the comparison of temperature profiles for different grid sizes of 8x8, 16x16 and 32x32. It can be concluded easily that the results of the code are grid independent. Fig. 5 shows the comparison of the Local Nusselt Numbers calculated from the temperature profiles, obtained from the code with that of Lee et al. [1]. It can be seen that the agreement between the code and Lee et al. results is reasonable and the difference is even less than 5%. Lee et al. has used central difference scheme at lower velocity and upwind scheme at relatively higher velocity. In the present study, Power Law Scheme has been used which is same as central difference scheme at lower velocity but differs from the upwind scheme until velocities are not too high. That is why; difference in code and Lee et al results has increased as the velocity increases along the length of the cylinder. There are also some other inaccuracies attached with the numerical methods and the disagreement of this magnitude can be tolerated.

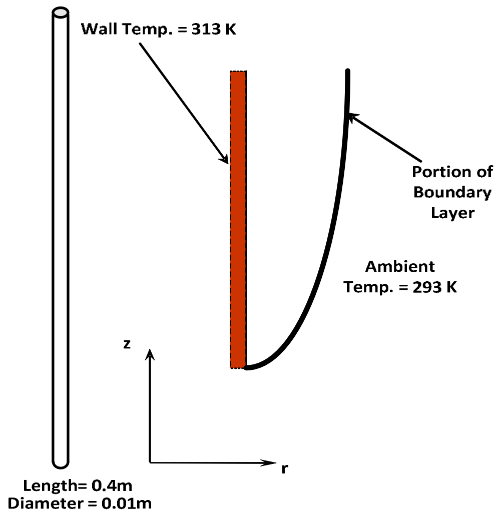

Figure 1: Schematic Diagram of a Vertical Cylinder in Infinite Medium with Const. Wall Temp.

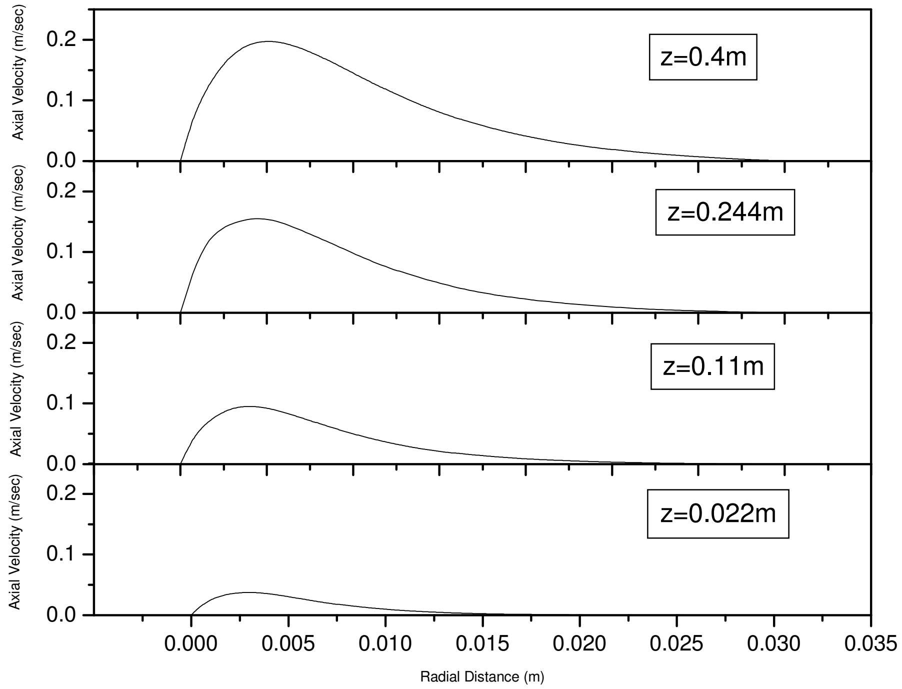

Figure 2: Velocity profiles at different z locations

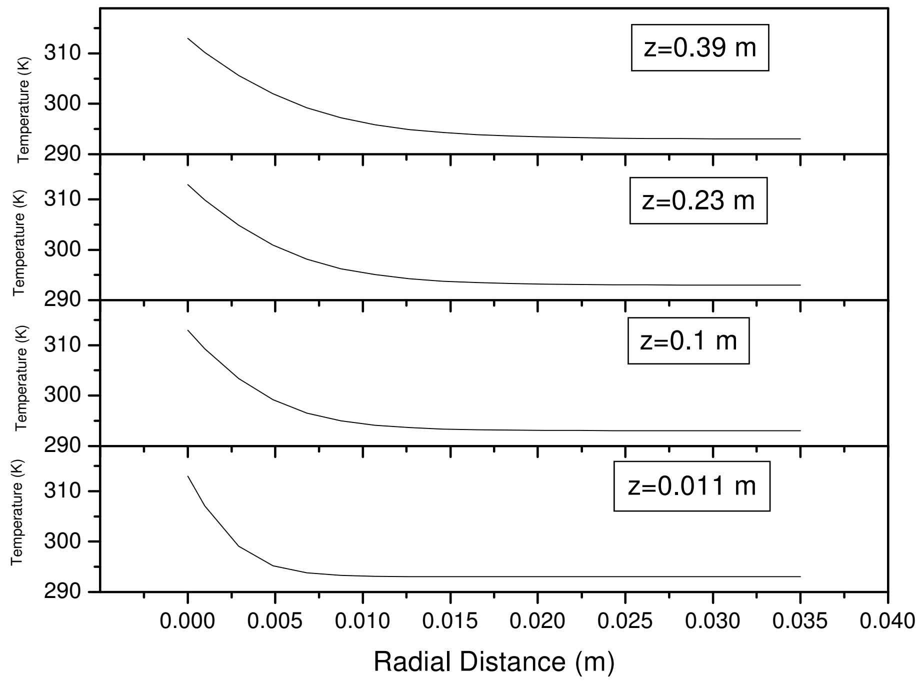

Figure 3: Temperature profiles at different z locations

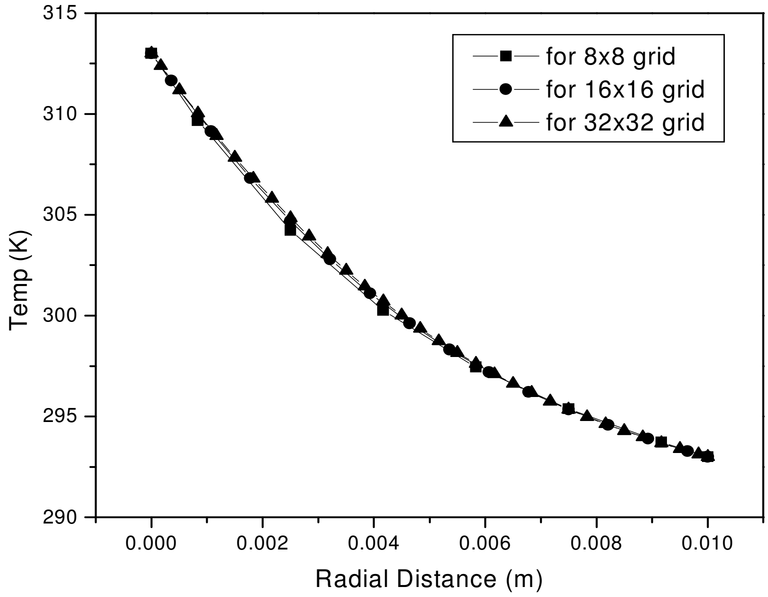

Figure 4: Comparison at different grid sizes

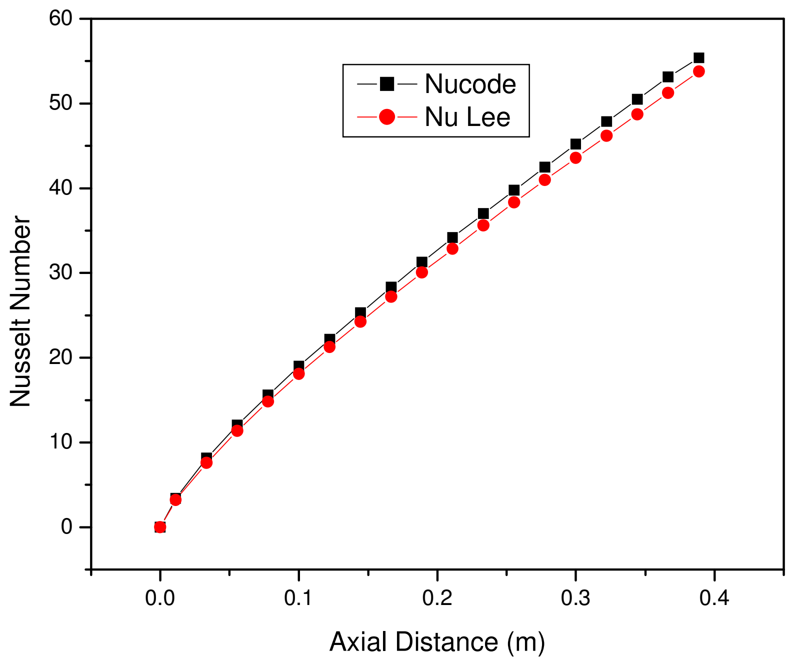

Figure 5: Comparison of Nu with theory

4 NATURAL CONVECTION HEAT TRANSFER ALONG A FLAT PLATE IN AN INFINITE MEDIUM (CASE 2)

For fluids having a Prandtl number 0.7 and higher, a vertical cylinder may be treated as a vertical flat plate provided that the following criterion is met:

$$\frac{L/D}{(Gr_L)^{1/4}} < 0.025 \tag{4}$$

Where D is the diameter of the cylinder, Gr_L is the local Grashof number and L is the length of the cylinder.

4.1.1 CONSTANT WALL TEMPERATURE

Jaluria [3] has solved the problem of natural convection from vertical flat plate using method of similarities. An expression for local Nusselt number has been suggested by Jaluria is:

$$Nu_x = 0.39(Gr_x \Pr)^{1/4} \tag{5}$$

Where Nu_x and Gr_x are the local Nusselt and Grashof numbers at a distance x from the lower end of the hot plate. The above expression is valid for laminar natural convection only i.e. for $10^4 < Ra < 10^9$. For simulation purpose, thin vertical cylinder is given a larger inner radius of 5m and outer radius of 5.01m. The height of the plate is assumed to be 0.2m. The problem is represented schematically in Fig.6.

4.1.1.1 BOUNDARY CONDITIONS

Air at atmospheric pressure has been used as the surrounding heat transfer medium and its temperature is assumed to be 293 K. The wall temperature has been assumed to be 313 K such that the temperature difference between the wall and surroundings is 20 K. For this temperature difference, the Rayleigh number for the geometry under consideration remains less than 10^8. The critical value of the Ra at which transition from laminar to turbulent occurs has been reported for this geometry by Ozisik [5] to be 10^9. Hence, a laminar flow model has been assumed. The mesh size of 20x20 is taken to simulate the current problem.

4.2 RESULTS AND DISCUSSION

Fig. 7 shows the u-velocity and Fig. 8 shows the temperature profiles for the case of the vertical flat plate. In the Fig. 9, it can be seen that the results of the code, when a vertical cylinder has been approximated as a vertical flat plate, are grid independent.

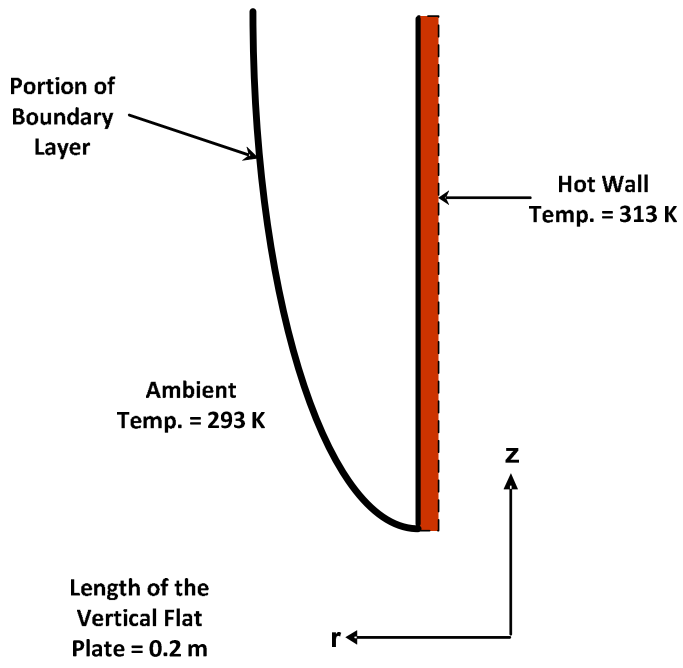

Figure 6: A Portion of Boundary Layer for a Vertical Flat Plate in Infinite Medium

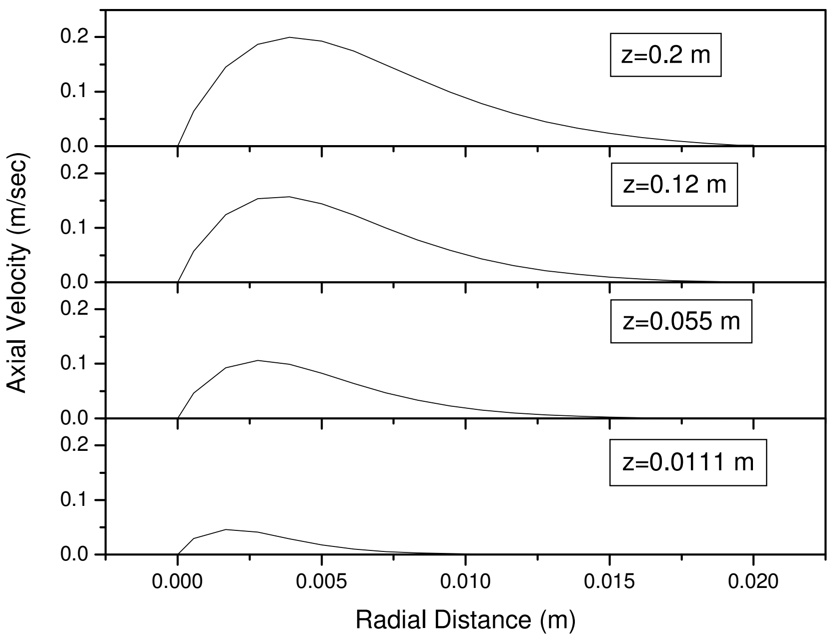

Figure 7: Velocity profile for 20x 20 grid for flat plate

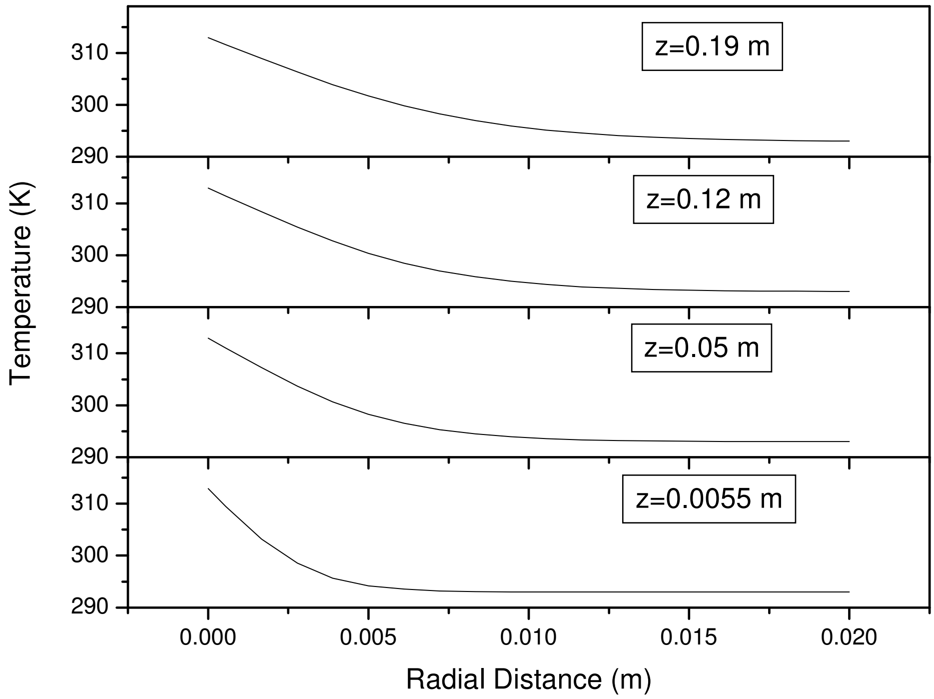

Figure 8: Temperature profile for 20x20 grid for flat plate

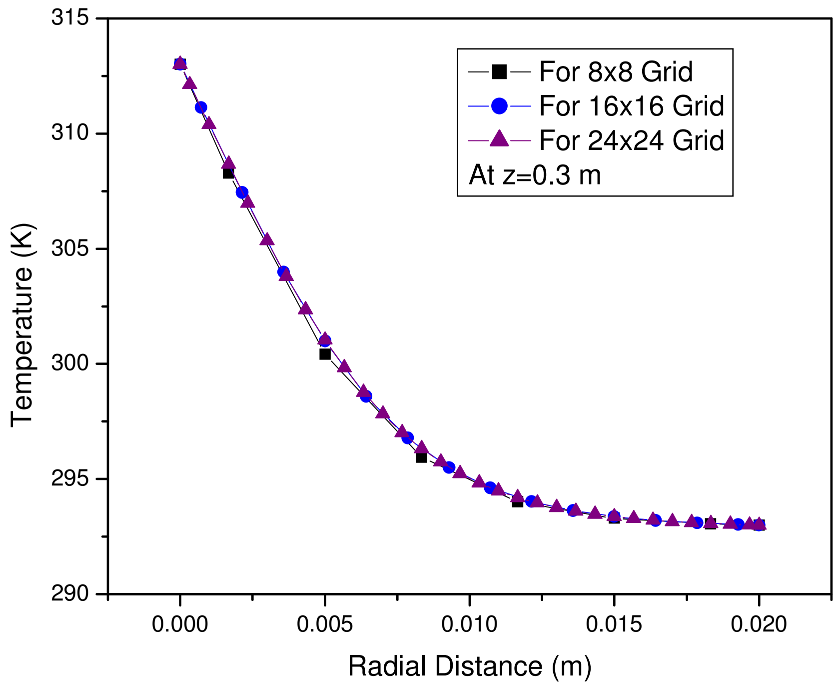

Figure: 9 Grid independence for the case of flat plate

As mentioned earlier, Jaluria [3] has presented an analytical solution for the local heat transfer coefficient from vertical flat plate. This expression was derived using Boussinesq approximation. The local heat transfer coefficient given by Eq.5 has been compared with the local heat transfer coefficient calculated numerically using the power law scheme. The agreement between the analytical and numerical results is reasonable. The numerical results under predict the heat transfer coefficient found by Eq.5 by less than 5%. The Fig.10 shows that maximum mismatch occurs in the end where the velocity profile becomes almost negligible. This discrepancy may be due to the fact that the solution presented by Jaluria utilizes some simplifying assumptions whereas the present method does not involve any such assumption. Moreover, approximations involved in the numerical methods also lead to the errors in the results. The relation used for the calculation of local heat transfer coefficient numerically is the following

$$q'' = h_x (T_w - T_\infty) \quad (6)$$

Where h_x is the local heat transfer coefficient and q'' is the heat flux calculated by the following expression

$$q'' = -k \frac{\partial T}{\partial r} \quad (7)$$

So for calculating the heat flux derivative of the temperature is required. This derivative of the temperature has been calculated using the straight-line approximation for the temperature profile, which is not the case here. Actually, temperature profile has a curve. Also for accurate value of q'', the value of the temperature is required at the very next point to the wall. But from code results, the next value of the temperature is obtained at the grid point, which is at a finite distance from the wall. So these factors also contribute to the difference between the analytical and numerical results. The comparison between the analytical and numerical results is shown in Fig.10.

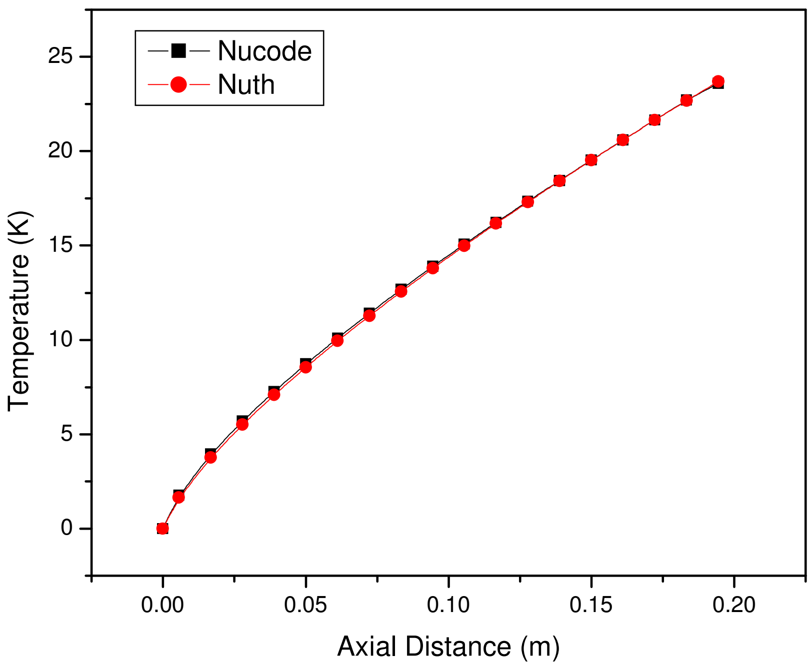

Figure 10: Nu Comparison with theory

5 CYLINDER WITH CONSTANT HEAT FLUX (CASE 3)

Natural convection from electrical conductors and nuclear fuel rods are typical examples of constant heat flux boundary condition on the wall. Hackle et al. [2] considered this type of boundary condition and suggested a numerical scheme to solve the governing equation for the case of thin cylinder. The correlation suggested by Hackle has been modified for a constant heat flux condition in this section. The modified correlation is given by:

$$Nu_X \, Gr_X{}^{*-1/5} = \alpha(Pr)\,[A(s) + f_1(Pr)s] \qquad (8)$$

The cylinder used in this section is the same as the one used for the case study of isothermal boundary condition Fig.11. Mesh as well as the boundary conditions used is also the same as in the previous case except a constant heat flux of 50 W/m^2 has been used on the wall instead of a constant wall temperature. Comparison of the Nusselt Number obtained from the results of the code is done with that of obtained from Eq. (8). This comparison has been shown in Fig 15. It should be noted the mesh independence study is not repeated for this case since the geometry is the same that of isothermal case.

5.1 RESULTS AND DISCUSSION

Fig 12 and Fig 13, show the u-velocity and temperature profiles for the case of a vertical cylinder with constant heat flux of 50 W/m^2 applied on its wall. A notable aspect of the natural convection from a cylinder with constant heat flux is that the wall temperature increases along the axial distance. It is because of the fact that the heat transfer coefficients decrease in the upper portion of the cylinder; hence, the temperature of the wall has to increase to deliver the same heat per unit area, as does the lower portion. This increase of wall temperature can be seen in Fig 14**.**

Fig 5-18 shows the comparison of the Local Nusselt Numbers obtained by the code temperature profiles with those calculated by Eq. 8. It can be seen that the agreement between the two results is quite good. In view of the fact that Hackle et al. [2] presented a correlation that had an average deviation of 5% from the actual numerical data, the small discrepancy between the present results and that of Heckel et al. [2] is not unexpected.

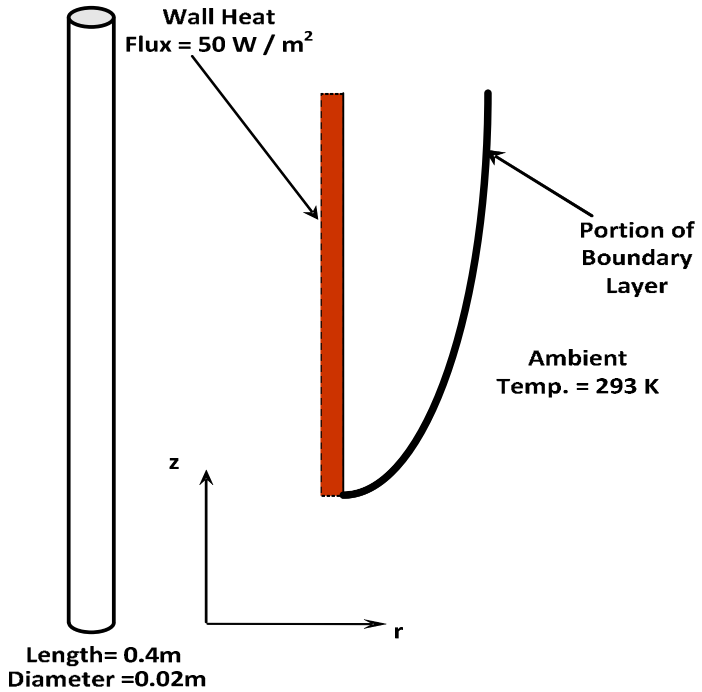

Figure 11: Schematic Diagram of a Vertical Cylinder in Infinite Medium With Const. Heat Flux.

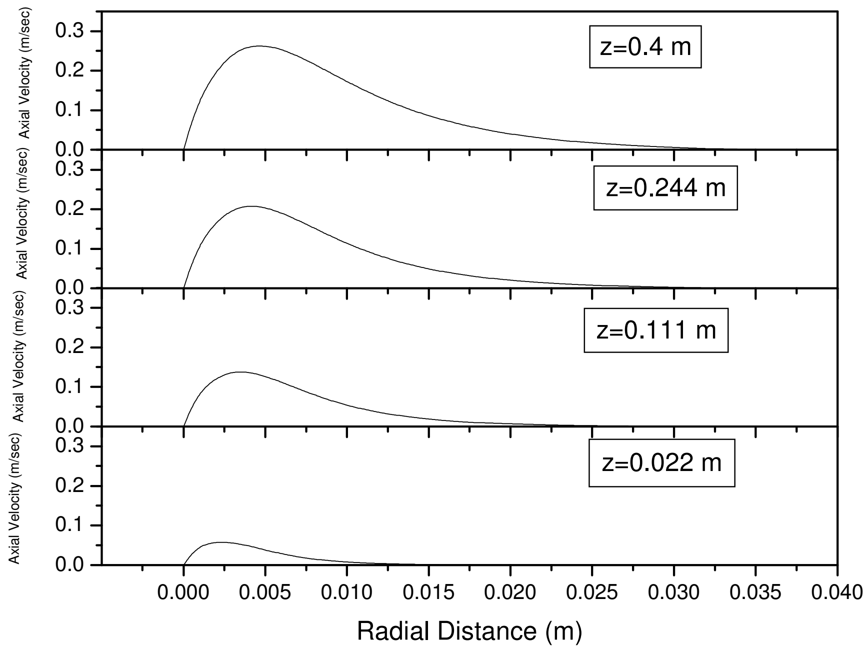

Figure 12: u-Velocity Profiles For 20x20 Grid For Constant Heat Flux

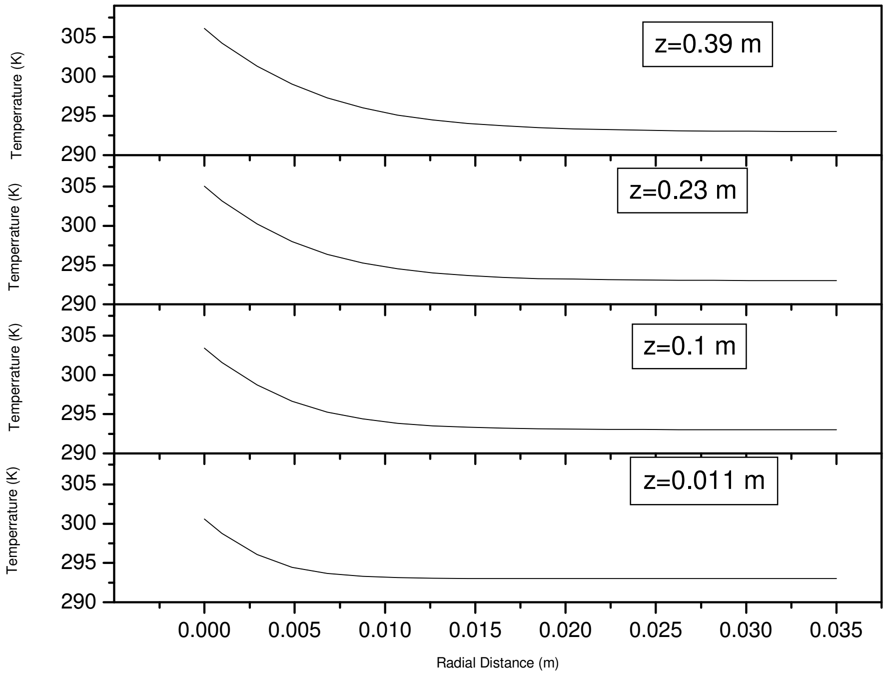

Figure 13: Temperature Profiles for 20 x 20 Grid For Constant Heat Flux

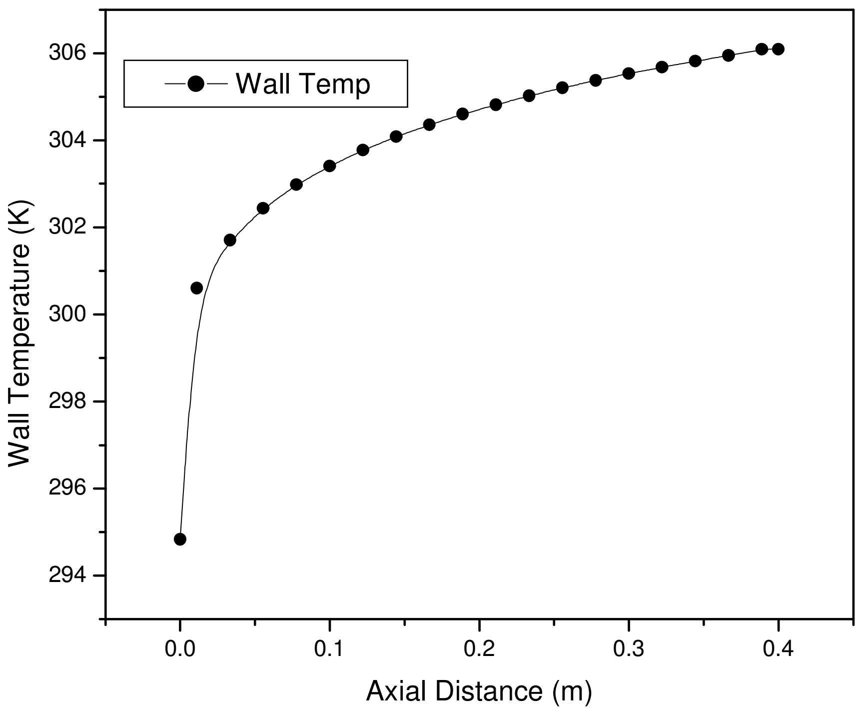

Figure 14: Wall Temperature along a vertical cylinder with constant heat flux

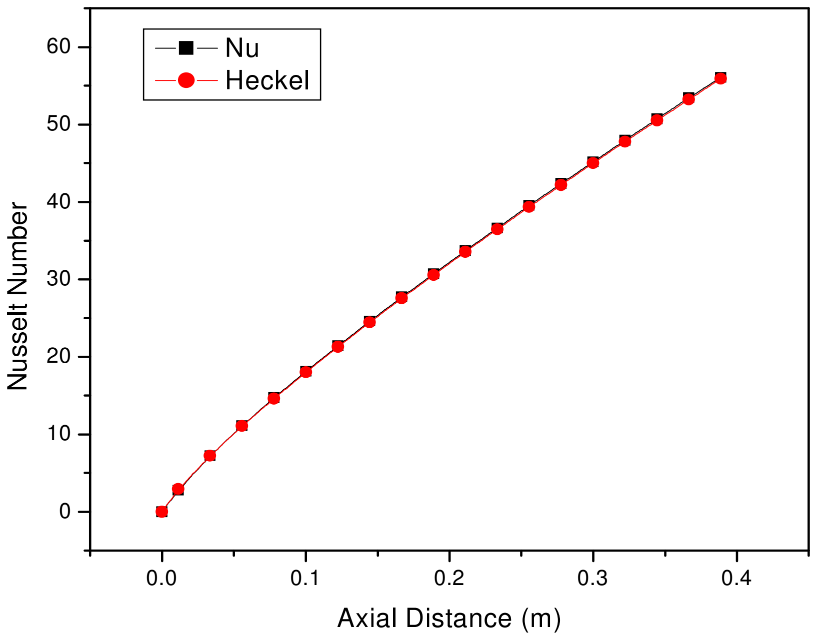

Figure 15: Comparison of Nu From Code And Hackle et al. [2]

6 NATURAL CONVECTION IN AN OPEN REACTANGULAR PITCH ASSEMBLY OF CYLINDERS

Three-dimensional simulation of an assembly of cylinders is an expensive computational task. Large memory requirements and long computational times are required to carry out such simulations. In many practical problems this approach may still not be used due to excessive computational power required and hence some sort of simplifications have to be made to solve the problem in reasonable time and memory requirements. Davis and Perona [21] have suggested a numerical method to study natural convection in an open triangular pitch assembly of vertical cylinders. They pointed out that in such an assembly a hexagon could be circumscribed around each cylinder, to represent the flow area and fluid assignable to the cylinder. They further simplified the problem by assuming that replacing the hexagon with a circle of equal area can approximate this situation. Based on this assumption, they have presented an axisymetric non-CFD analysis of such a cylinder for various pitches to diameter ratios.

In this section, the idea of Davis and Perona [21] has been used to simulate natural convection heat transfer in an assembly of vertical cylindrical fuel rods laid on a rectangular pitch in a spent fuel bay. For rectangular pitch a rectangle is circumscribed, instead of a hexagon, around the cylinder. Also, the assumption of representing this rectangle with an equal area circle has been made. The data for this analysis has been used for typical PWR fuel rods in spent fuel bay. The height of the cylinder used is 2.9m and radius is 0.00538m (see Figure 16).

6.1 BOUNDARY CONDITIONS

The thermal boundary condition applied to the cylinder is a constant wall heat flux of 22.182 W/m^2. The top and bottom ends of the domain have been considered as constant pressure boundaries with static temperatures of 300 K. On the outer hypothetical circle, assumed for the circumscribed rectangle, symmetry boundary has been assumed.

Symmetry boundary condition means that component of the gradient normal to the plane should be zero. Geometry is the mirror image across this boundary and on these boundaries the normal velocity is taken as zero.

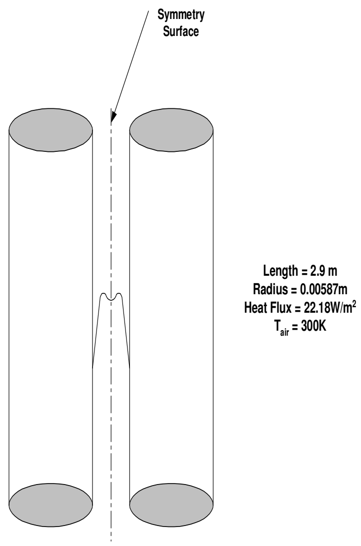

Figure 16: Fuel rod assembly.

6.2 RESULTS AND DISCUSSION

A 20x 20 mesh has been used to solve the problem. The u-Velocity profiles for this case have been shown in Figure 17. The u- velocity increases as distance from the wall increases and then it reaches the maximum near the symmetry plane. The increase in the velocity is due to the buoyancy force. Although the buoyancy force is the highest near the wall but due to wall effects, velocity is zero there. As we move away from wall, buoyancy force decreases, as the temperature decreases, even then the force is enough to cause a continuous increase in u-velocity as the gap is small and temperature decrease is very small. A mirror image of the profile is there across the symmetry boundary. The temperature profiles have been shown in Fig 18. They are the usual temperature profiles as mostly found in the case of natural convection. Fig 19 shows the heat transfer coefficient along the length of the fuel rod. It shows a decreasing trend with length. It is due to the fact that along the length of the fuel rod as the boundary layer develops, the thermal resistance increases causing a decrease in the heat transfer coefficient and hence in the heat transfer.

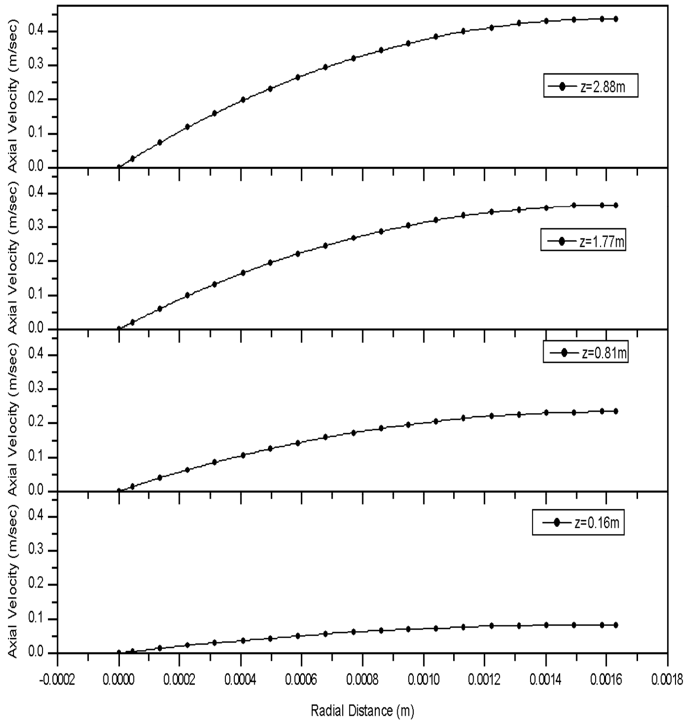

Figure 17: u-Velocity profiles for 20x20 grid for vertical cylindrical assembly

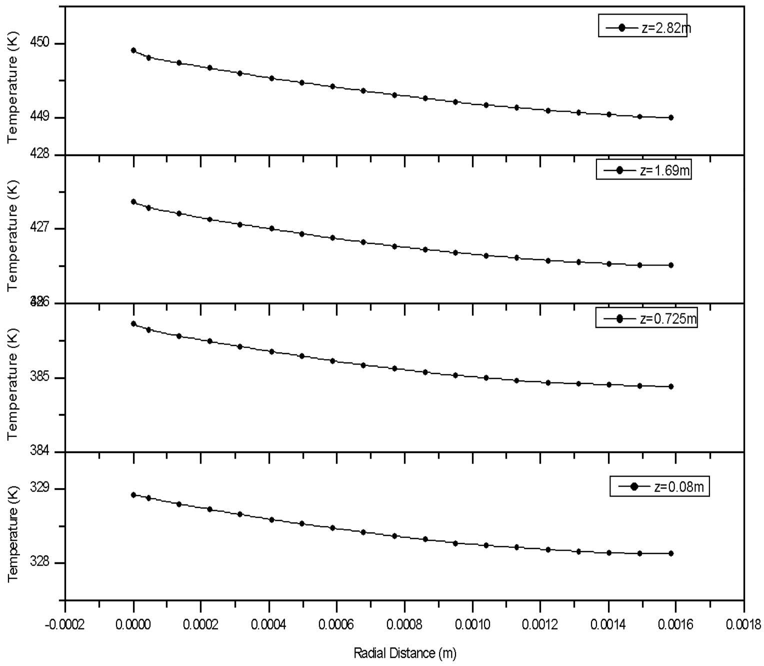

Fig 18: Temperature profiles for 20x20 grid for vertical cylindrical assembly

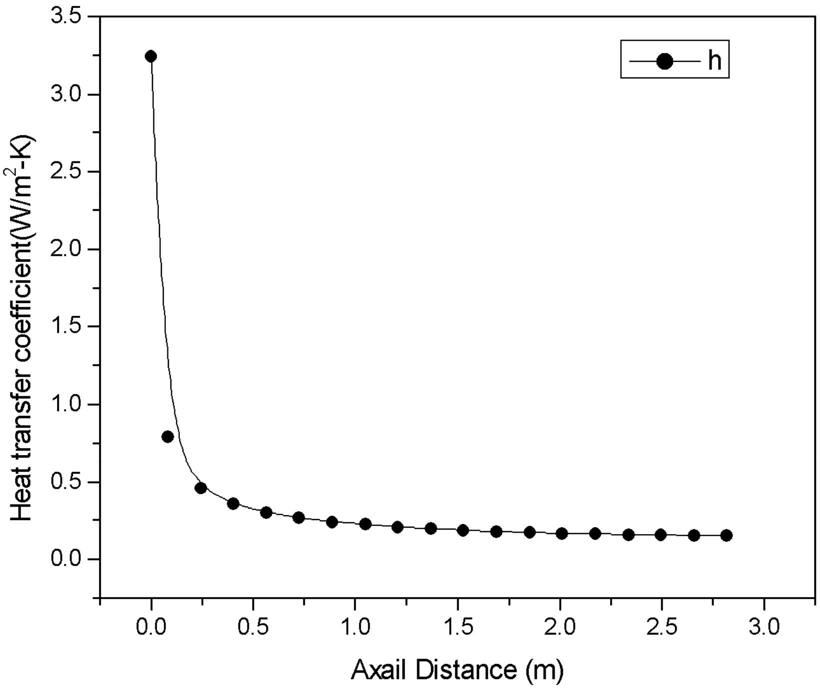

Figure 19: Heat transfer Coefficient Along the Length of Fuel Rod

7 CONCLUSION

A two-dimensional computer code in FORTRAN, for a vertical cylinder subjected to natural convection has been developed. For this purpose, momentum, continuity and energy equations for vertical cylinder in cylindrical coordinates have been solved. The flow regime in all the case studies has been assumed to be Laminar. The natural convection has been studied for the following geometries:

- From vertical cylinder in infinite medium
- From a vertical flat plate in an infinite medium.
- From an open assembly of finite vertical cylinders
- From an open rectangular pitch assembly of cylinders

For a vertical cylinder in infinite medium, two types of thermal boundary conditions have been used i.e. a constant wall temperature of 313 K and a constant heat flux on the wall of 50 W/m^2 .The u- velocity and temperature profiles have been presented. The calculated Local Nusselt numbers have been compared with results from the Lee et al and Hackle et al. The agreement between the present study and the previous work has been found to be within reasonable range. For the case of a vertical flat plate the calculated local Nusselt numbers have been compared with the available empirical correlation. It is found that the calculated values of local Nusselt numbers fall well within those obtained by empirical correlation. Natural convection from an open assembly of infinite number of cylinders with constant heat flux has been studied for Pitch to Diameter ratio of 1.33**.**

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